如何（不）实行价格歧视？——基于一般消费者分布的二维模型分析

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在线附录

附录 I.　引理 1 的证明

这是 Liu 和 Shuai（2019）研究中的一个特例，其中$t_1 = t_2 = t$（见其命题 1）。为完整起见，本文给出以下证明。

设 p_A 和 p_B 分别表示 A 企业和 B 企业的价格。根据 $p_A \le p_B$ 或 $p_A > p_B$，存在两种需求结构。这两种结构是相似的，因此不失一般性地假设 $p_A \le p_B$。

设$(x,\ \bar{y})$为代表性边际消费者。根据正文第 5 页的方程（4），得到

$$\bar{y} = \frac{p_B - p_A + 2t - 2tx}{2t}. \tag{1}$$

设$(x_1, 1)$和$(1, y_1)$分别为两个边际消费者且

$$x_1 = y_1 = \frac{p_B - p_A}{2t}. \tag{2}$$

企业利润为

$$\pi_A = p_A q_A\,, \pi_B = p_B q_B, \tag{3}$$

其中

$$q_B = \int_{x_1}^{1} [1 - F_2(\bar{y})] f_1(x) dx\,, q_A = 1 - q_B. \tag{4}$$

注意到

$$\frac{\partial q_A}{\partial p_A} = \frac{\partial q_B}{\partial p_B} = -\frac{1}{2t} \int_{x_1}^{1} f_2(\bar{y}) f_1(x) dx, \tag{5}$$

一阶条件为

$$\frac{\partial \pi_A}{\partial p_A} = q_A - \frac{p_A}{2t} \int_{x_1}^{1} f_2(\bar{y}) f_1(x) dx = 0, \tag{6}$$

$$\frac{\partial \pi_B}{\partial p_B} = q_B - \frac{p_B}{2t} \int_{x_1}^{1} f_2(\bar{y}) f_1(x) dx = 0, \tag{7}$$

联合可得

$$\frac{q_A}{q_B} = \frac{p_A}{p_B}. \tag{8}$$

上文已经假设了 $p_A \ge p_B$。如果 $p_A > p_B$，则边际消费者线将位于角点(0, 1)的下方。那么 $q_A < q_B$，且上述等式无法成立。①因此，必然有

$$p_A = p_B \Longrightarrow x_1 = 0,\ \bar{y} = 1 - x\,,\ q_A = q_B = \frac{1}{2}, \tag{9}$$

这样均衡统一定价为

$$p^{U-U} = \frac{t}{\int_0^1 f_2(1-x) f_1(x)}, \tag{10}$$

每个企业的利润为

$$\pi^{U-U} = \frac{p^{U-U}}{2} = \frac{t}{2\int_0^1 f_2(1-x) f_1(x)}. \tag{11}$$

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①如果假设$p_A > p_B$，也会得出类似的结论。

附录II.　　引理 3 的证明

这只针对第 1 子市场，即[0, 1/2] × [0, 1]中的消费者。需要证明在(*D-D1*)中，情形 2：$p_{1A} \geq p_{1B} + t$ 和情形 3：$p_{1A} \leq p_{1B}$ 不能作为均衡存在。根据引理 2，只需证明一阶条件的解在情形 2 和情形 3 中仍然推出$\frac{q_{1A}}{q_{1B}} = \frac{p_{1A}}{p_{1B}}$。

设$(x, \bar{y})$为代表性边际消费者，其中

$$\bar{y} = \frac{p_{1B}-p_{1A}+2t-2tx}{2t}, \tag{12}$$

那么

$$\frac{\partial \bar{y}}{\partial p_{1A}} =-\frac{1}{2t}, \frac{\partial \bar{y}}{\partial p_{1B}} = \frac{1}{2t}. \tag{13}$$

情形 2：$p_{1A} \geq p_{1B} + t$

设$(x_1, 0)$ 和 $(0, y_1)$ 分别为两个边际消费者，其中

$$x_1 = \frac{p_{1B}-p_{1A}+t}{2t}, y_1 = \frac{p_{1B}-p_{1A}+t}{2t}, \tag{14}$$

企业利润为

$$\pi_{1A} = p_{1A}q_{1A}, \pi_{1B} = p_{1B}q_{1B}, \tag{15}$$

其中

$$q_{1A} = \int_0^{x_1} F_2(\bar{y}) f_1(x) dx, \ q_{1B} = \frac{1}{2} - q_{1A}. \tag{16}$$

由$F_2(\bar{y}(x_1)) = F_2(0) = 0$，可得

$$\frac{\partial q_{1A}}{\partial p_{1A}} = \frac{\partial q_{1B}}{\partial p_{1B}} =-\frac{1}{2t}\int_0^{x_1} f_2(\bar{y}) f_1(x) dx, \tag{17}$$

一阶条件为

$$\frac{\partial \pi_{1A}}{\partial p_{1A}} = q_{1A} - \frac{p_{1A}}{2t}\int_0^{x_1} f_2(\bar{y}) f_1(x) dx = 0, \tag{18}$$

$$\frac{\partial \pi_{1B}}{\partial p_{1B}} = q_{1B} - \frac{p_{1B}}{2t}\int_0^{x_1} f_2(\bar{y}) f_1(x) dx = 0, \tag{19}$$

联合可得

$$\frac{q_{1A}}{q_{1B}} = \frac{p_{1A}}{p_{1B}}. \tag{20}$$

情形 3：$p_{1A} \leq p_{1B}$

在该情形下，边际消费者是$(x_1, 1)$和$(1/2, y_1)$，其中

$$x_1 = \frac{p_{1B}-p_{1A}}{2t}, y_1 = \frac{p_{1B}-p_{1A}}{2t} + \frac{1}{2}, \tag{21}$$

企业利润为

$$\pi_{1A}=p_{1A}q_{1A}\,,\pi_{1B}=p_{1B}q_{1B}, \tag{22}$$

其中

$$q_{1B}=\int_{x_1}^{1/2}[1-F_2(\bar{y})]f_1(x)dx\,,q_{1A}=\frac{1}{2}-q_{1B}. \tag{23}$$

由$F_2(\bar{y}(x_1))=F_2(1)=1$，可得

$$\frac{\partial q_{1A}}{\partial p_{1A}}=\frac{\partial q_{1B}}{\partial p_{1B}}=-\frac{1}{2t}\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx, \tag{24}$$

一阶条件为

$$\frac{\partial \pi_{1A}}{\partial p_{1A}}=q_{1A}-\frac{p_{1A}}{2t}\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx=0, \tag{25}$$

$$\frac{\partial \pi_{1B}}{\partial p_{1B}}=q_{1B}-\frac{p_{1B}}{2t}\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx=0, \tag{26}$$

联合可得

$$\frac{q_{1A}}{q_{1B}}=\frac{p_{1A}}{p_{1B}}. \tag{27}$$

附录III.　　命题 1 的证明

已有

$$\bar{y}=\frac{p_{1B}-p_{1A}+2t-2tx}{2t},\tag{28}$$

以及

$$\frac{\partial\bar{y}}{\partial p_{1A}}=-\frac{1}{2t},\frac{\partial\bar{y}}{\partial p_{1B}}=\frac{1}{2t}.\tag{29}$$

企业的需求函数为

$$q_{1A}=\int_0^{1/2}F_2(\bar{y})f_1(x)dx,q_{1B}=\frac{1}{2}-q_{1A},\tag{30}$$

可得

$$\frac{\partial q_{1A}}{\partial p_{1A}}=\frac{\partial q_{1B}}{\partial p_{1B}}=-\frac{1}{2t}\int_0^{1/2}f_2(\bar{y})f_1(x)dx,\tag{31}$$

企业利润为

$$\pi_{1A}=p_{1A}q_{1A},\pi_{1B}=p_{1B}q_{1B},\tag{32}$$

一阶条件为

$$\frac{\partial\pi_{1A}}{\partial p_{1A}}=q_{1A}-\frac{p_{1A}}{2t}\int_0^{1/2}f_2(\bar{y})f_1(x)dx=0,\tag{33}$$

$$\frac{\partial\pi_{1B}}{\partial p_{1B}}=q_{1B}-\frac{p_{1B}}{2t}\int_0^{1/2}f_2(\bar{y})f_1(x)dx=0.\tag{34}$$

根据假设 1，该博弈中存在价格上的纯策略均衡（Caplin 和 Nalebuff）。因为只在这种情形下才存在均衡，所以方程（33）和（34）必定有解。通过这两个方程，可以得到

$$\frac{q_{1A}}{q_{1B}}=\frac{p_{1A}}{p_{1B}}.\tag{35}$$

因为 $p_{1A}>p_{1B}$，所以进一步得到$q_{1A}>\frac{1}{4}>q_{1B}$。由于对称性，$A$ 企业从第 2 子市场获得的利润与 B 企业从第 1 子市场获得的利润相同，即$\pi_{2A}=\pi_{1B}$。因此，在均衡状态下，每个企业的利润为

$$\pi^{D-D1}=\pi_{1A}+\pi_{1B}=p_{1A}q_{1A}+p_{1B}q_{1B}.\tag{36}$$

将方程（33）和（34）相加，可以得到

$$\frac{1}{2}-\frac{p_{1A}+p_{1B}}{2t}\int_0^{1/2}f_2(\bar{y})f_1(x)dx=0,\tag{37}$$

可得

$$p_{1A}+p_{1B}=\frac{t}{\int_0^{1/2} f_2(\bar{y})f_1(x)dx}$$

$$=\frac{t}{\int_0^{1/2} f_2(\frac{p_{1B}-p_{1A}+2t-2tx}{2t})f_1(x)dx}. \quad (38)$$

令 $p_{1A}-p_{1B}=k\in(0,\ t)$。那么当 $k=0$ 时，$\frac{-k+2t-2tx}{2t}=1-x$。在条件 2 下，$\frac{\partial \int_0^{1/2} f_2(\frac{-k+2t-2tx}{2t})f_1(x)dx}{\partial k}=\int_0^{1/2} f_2^{'}(\frac{-k+2t-2tx}{2t})(-\frac{1}{2t})f_1(x)dx\le 0,\forall k\in(0,t).$

可得

$$\int_0^{1/2} f_2(\frac{-k+2t-2tx}{2t})f_1(x)dx\,,\forall k\in(0,t)\le\int_0^{1/2} f_2(1-x)f_1(x)dx\ \ (k=0), \quad (39)$$

进一步表示为

$$p_{1A}+p_{1B}\ge 2p^{U-U}. \quad (40)$$

已有$p^{U-U}=\frac{t}{\int_0^1 f_2(1-x)f_1(x)dx}=\frac{t}{2\int_0^{1/2} f_2(1-x)f_1(x)dx}$，那么

$$\begin{aligned}\pi^{D-D1}&=p_{1A}q_{1A}+p_{1B}q_{1B}\\&>p_{1A}\cdot\frac{1}{4}+p_{1B}\cdot\frac{1}{4}\\&=\frac{1}{4}\cdot(p_{1A}+p_{1B})\\&>\frac{1}{4}\cdot 2p^{U-U}\\&=\pi^{U-U}.\end{aligned} \quad (41)$$

附录IV.　　引理 4 的证明

设$(x,\ \bar{y})$为边际消费者线上的代表性消费者，其中

$$\bar{y}=\frac{p_{1B}-p_{1A}+2t-2tx}{2t}\,, \tag{42}$$

设$(x_1, 1/2)$和$(1/2, y_1)$分别为两个边际消费者，其中

$$x_1=y_1=\frac{p_{1B}-p_{1A}+t}{2t}, \tag{43}$$

有

$$\frac{\partial\bar{y}}{\partial p_{1A}}=\frac{\partial x_1}{\partial p_{1A}}=-\frac{1}{2t}\,,\ \frac{\partial\bar{y}}{\partial p_{1B}}=\frac{\partial x_1}{\partial p_{1B}}=\frac{1}{2t}. \tag{44}$$

第 1 子市场中企业的需求函数为

$$q_{1A}=\frac{1}{2}F_1(x_1)+\int_{x=x_1}^{1/2}F_2(\bar{y})f_1(x)dx\,,\ q_{1B}=\int_{x=x_1}^{1/2}[1/2-F_2(\bar{y})]f_1(x)dx, \tag{45}$$

第 1 子市场中企业利润为

$$\pi_{1A}=p_{1A}q_{1A}\,,\ \pi_{1B}=p_{1B}q_{1B}. \tag{46}$$

一阶条件为（有 $\mathrm{F}(\bar{y}(x_1)=1/2)$），

$$\frac{\partial\pi_{1A}}{\partial p_{1A}}=q_{1A}+p_{1A}\left(\frac{1}{2}f_1(x_1)\frac{\partial x_1}{\partial p_{1A}}-F_2(\bar{y}(x_1))f_1(x_1)\frac{\partial x_1}{\partial p_{1A}}+\int_{x=x_1}^{1/2}f_2(\bar{y})\frac{\partial\bar{y}}{\partial p_{1A}}f_1(x)dx\right)$$

$$=q_{1A}+p_{1A}\left(-\frac{f_1(x_1)}{4t}+\frac{f_1(x_1)}{4t}-\frac{1}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx\right)$$

$$=q_{1A}-\frac{p_{1A}}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx=0, \tag{47}$$

$$\frac{\partial\pi_{1B}}{\partial p_{1B}}=q_{1B}+p_{1B}\left(-[1/2-F_2(\bar{y}(x_1))]f_1(x_1)\frac{\partial x_1}{\partial p_{1B}}+\int_{x=x_1}^{1/2}-f_2(\bar{y})\frac{\partial\bar{y}}{\partial p_{1B}}f_1(x)dx\right)$$

$$=q_{1B}+p_{1B}\left(0-\frac{1}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx\right)$$

$$=q_{1B}-\frac{p_{1B}}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx=0. \tag{48}$$

根据方程（47）和（48），可得

$$q_{1A}+q_{1B}=\frac{p_{1A}+p_{1B}}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx$$

$$\Longrightarrow\ \frac{1}{4}=\frac{p_{1A}+p_{1B}}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx$$

$$\Longrightarrow\ p_{1A}+p_{1B}=\frac{t}{2\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx}$$

$$q_{1A}-q_{1B}=\frac{p_{1A}-p_{1B}}{2t}\int_{x=x_1}^{1/2}f_2(\bar{y})f_1(x)dx. \tag{49}$$

设$k_1=p_{1A}-p_{1B}$，那么$x_1=\frac{t-k_1}{2t}$。第 1 子市场的联合利润为

$$
\begin{aligned}
\pi_1 &= p_{1A}q_{1A} + p_{1B}q_{1B} \\
&= \frac{(p_{1A}+p_{1B})(q_{1A}+q_{1B})}{2} + \frac{(p_{1A}-p_{1B})(q_{1A}-q_{1B})}{2} \\
&= \frac{p_{1A}+p_{1B}}{8} + k_1\frac{q_{1A}-q_{1B}}{2} \\
&= \frac{t}{16\int_{x=x_1}^{1/2} f_2(\bar{y})f_1(x)dx} + \frac{{k_1}^2}{4t}\int_{x=x_1}^{1/2} f_2(\bar{y})f_1(x)dx \\
&= \pi_1(k_1).
\end{aligned}
\qquad (50)
$$

设p_{1A}^*和p_{1B}^*为方程（47）和（48）的解，且$k^* = p_{1A}^* - p_{1B}^*$。当且仅当

$$
\pi_1(k_1^*) < \frac{t}{4\int_{x=0}^{1/2} f_2(1-x)f_1(x)dx} \ , \qquad (51)
$$

即当满足条件 3 时，价格歧视（*DD-DD*）会降低第 1 子市场企业的联合利润。

附录V.　　引理 5 的证明

$$\pi_1(k_1^*) < \frac{t}{4\int_{x=0}^{1/2} f_2(1-x) f_1(x)dx}. \quad (51)$$

设$(x,\ \bar{y})$为边际消费者线上的代表性消费者，其中

$$\bar{y} = \frac{p_{2B} - p_{2A} + 2t - 2tx}{2t}, \quad (52)$$

设$(x_2, 1)$和$(1/2, y_2)$为两个边际消费者，其中

$$x_2 = \frac{p_{2B} - p_{2A}}{2t},\ y_2 = \frac{p_{2B} - p_{2A} + t}{2t}, \quad (53)$$

有

$$\frac{\partial \bar{y}}{\partial p_{2A}} = \frac{\partial x_2}{\partial p_{2A}} = -\frac{1}{2t},\ \frac{\partial \bar{y}}{\partial p_{2B}} = \frac{\partial x_2}{\partial p_{2B}} = \frac{1}{2t}. \quad (54)$$

第 2 子市场的企业需求函数为：

$$q_{2A} = \frac{1}{2}F_1(x_2) + \int_{x=x_2}^{1/2} [F_2(\bar{y}) - 1/2] f_1(x)dx,\ q_{2B} = \int_{x=x_2}^{1/2} [1 - F_2(\bar{y})] f_1(x)dx. \quad (55)$$

第 2 子市场企业利润为：

$$\pi_{2A} = p_{2A} q_{2A},\ \pi_{2B} = p_{2B} q_{2B}. \quad (56)$$

一阶条件为（有$F_2(\bar{y}(x_2) = 1)$），

$$\frac{\partial \pi_{2A}}{\partial p_{2A}} = q_{2A} + p_{2A}\left(\frac{1}{2}f_1(x_2)\frac{\partial x_2}{\partial p_{2A}} - [F_2(\bar{y}(x_2)) - 1/2] f_1(x_2)\frac{\partial x_2}{\partial p_{2A}} + \int_{x=x_2}^{1/2} f_2(\bar{y})\frac{\partial \bar{y}}{\partial p_{2A}} f_1(x)dx\right)$$

$$= q_{2A} + p_{2A}\left(-\frac{f_1(x_2)}{4t} + \frac{f_1(x_2)}{4t} - \frac{1}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx\right)$$

$$= q_{2A} - \frac{p_{2A}}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx = 0, \quad (57)$$

$$\frac{\partial \pi_{2B}}{\partial p_{2B}} = q_{2B} + p_{2B}\left(-[1 - F_2(\bar{y}(x_2))] f_1(x_2)\frac{\partial x_2}{\partial p_{2B}} + \int_{x=x_2}^{1/2} -f_2(\bar{y})\frac{\partial \bar{y}}{\partial p_{2B}} f_1(x)dx\right)$$

$$= q_{2B} + p_{2B}\left(0 - \frac{1}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx\right)$$

$$= q_{2B} - \frac{p_{2B}}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx = 0. \quad (58)$$

从方程（57）和（58）可以得到

$$q_{2A} + q_{2B} = \frac{p_{2A} + p_{2B}}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx,$$

$$\frac{1}{4} = \frac{p_{2A} + p_{2B}}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx,$$

$$p_{2A} + p_{2B} = \frac{t}{2\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx}, \quad (59)$$

以及

$$q_{2A} - q_{2B} = \frac{p_{2A} - p_{2B}}{2t}\int_{x=x_2}^{1/2} f_2(\bar{y}) f_1(x)dx. \quad (60)$$

设$k = p_{2A} - p_{2B}$。那么$x_2 = \frac{-k}{2t}$。第 2 子市场的联合利润为

$$\pi_2(k) = p_{2A} q_{2A} + p_{2B} q_{2B}$$

$$= \frac{(p_{2A}+p_{2B})(q_{2A}+q_{2B})}{2} + \frac{(p_{2A}-p_{2B})(q_{2A}-q_{2B})}{2}$$

$$= \frac{p_{2A}+p_{2B}}{8} + k\frac{q_{2A}-q_{2B}}{2}$$

$$= \frac{t}{16\int_{x=x_2}^{1/2} f_2(\bar{y})f_1(x)dx} + \frac{k^2}{4t}\int_{x=x_2}^{1/2} f_2(\bar{y})f_1(x)dx. \quad (61)$$

设p_{2A}^*和p_{2B}^*为方程（57）和（58）的解，且$k^* = p_{2A}^* - p_{2B}^*$。根据条件 4，可以得到

$$\pi_2^{DD-DD} < \frac{t}{4\int_{x=0}^{1/2} f_2(1-x)f_1(x)dx}. \quad (62)$$

也就是说，两个维度上的价格歧视会降低第 2 子市场企业的联合利润。

假设第 2 子市场消费者分布对称，在均衡状态下，p_{2A}和 p_{2B}接近，导致 k^* 接近零。在这种情形下，容易得到 p_{2A}和 p_{2B}接近 $p^{U\text{-}U}$的一半，而相对于统一定价，企业的联合利润必定下降。

附录VI.　　命题 5 的证明

由于对称性，本文只考虑了 A 企业的一阶条件。其需求函数为

$$q_{1A}=\frac{1}{4}-\int_{x_1}^{1/2}\left(\frac{1}{2}-F_2(\bar{y})\right)f_1(x)dx\,,q_{2A}=\frac{1}{4}-\int_{x_2}^{1/2}(1-F_2(\bar{y}))f_1(x)dx,$$

$$q_{3A}=\int_{1/2}^{x_3}\left(F_2(\bar{y})-\frac{1}{2}\right)f_1(x)dx\,,q_{4A}=\frac{1}{4}-\int_{x_4}^{1}\left(\frac{1}{2}-F_2(\bar{y})\right)f_1(x)dx. \tag{63}$$

在每个子市场中，$\bar{y}=\frac{p_B-p_A+2t-2tx}{2t}$，其中 p_A 和 p_B 分别为该子市场中 A 企业和 B 企业的价格。

然后通过计算得到以下导数：

$$\frac{\partial q_{1A}}{\partial p_{1A}}=-\frac{1}{2t}\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx\,,\ \frac{\partial q_{2A}}{\partial p_{1A}}=-\frac{1}{2t}\int_{x_2}^{1/2}f_2(\bar{y})f_1(x)dx,$$

$$\frac{\partial q_{3A}}{\partial p_{2A}}=-\frac{1}{2t}\int_{1/2}^{x_3}f_2(\bar{y})f_1(x)dx\,,\frac{\partial q_{4A}}{\partial p_{2A}}=-\frac{1}{2t}\int_{x_4}^{1}f_2(\bar{y})f_1(x)dx. \tag{64}$$

在对称性条件下（$p_{1A}=p_{2B}$ 和 $p_{2A}=p_{1B}$），通过计算得到 $x_3=1-x_1$ 和 $x_2=x_4=0$。
可得：

$$\frac{\partial q_{3A}}{\partial p_{2A}}=\frac{\partial q_{1A}}{\partial p_{1A}}\,,\frac{\partial q_{4A}}{\partial p_{2A}}=\frac{\partial q_{2A}}{\partial p_{1A}}\Longrightarrow\frac{\partial q_{3A}}{\partial p_{2A}}+\frac{\partial q_{4A}}{\partial p_{2A}}=\frac{\partial q_{1A}}{\partial p_{1A}}+\frac{\partial q_{2A}}{\partial p_{1A}}. \tag{65}$$

A 企业的利润为

$$\pi_A=p_{1A}(q_{1A}+q_{2A})+p_{2A}(q_{3A}+q_{4A}). \tag{66}$$

求解一阶导数并在对称处进行分析，得到

$$\frac{\partial\pi_A}{\partial p_{1A}}=(q_{1A}+q_{2A})+p_{1A}\left(\frac{\partial q_{1A}}{\partial p_{1A}}+\frac{\partial q_{2A}}{\partial p_{1A}}\right)=0$$

$$\Longrightarrow\ q_{1A}+q_{2A}=\frac{p_{1A}}{2t}\left(\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx+\int_0^{1/2}f_2(\bar{y})f_1(x)dx\right), \tag{67}$$

$$\frac{\partial\pi_A}{\partial p_{2A}}=(q_{3A}+q_{4A})+p_{2A}\left(\frac{\partial q_{3A}}{\partial p_{2A}}+\frac{\partial q_{4A}}{\partial p_{2A}}\right)=0$$

$$\Longrightarrow\ q_{3A}+q_{4A}=\frac{p_{2A}}{2t}\left(\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx+\int_0^{1/2}f_2(\bar{y})f_1(x)dx\right). \tag{68}$$

令$\Delta=\int_{x_1}^{1/2}f_2(\bar{y})f_1(x)dx+\int_0^{1/2}f_2(\bar{y})f_1(x)dx$，可以推出

$$\frac{1}{2}=(q_{1A}+q_{2A})+(q_{3A}+q_{4A})=\frac{p_{1A}+p_{2A}}{2t}\Delta\Longrightarrow p_{1A}+p_{2A}=\frac{t}{\Delta}$$

$$(q_{1A}+q_{2A})-(q_{3A}+q_{4A})=\frac{p_{1A}-p_{2A}}{2t}\Delta. \tag{69}$$

A 企业的利润为

$$\begin{aligned}\pi_A&=p_{1A}(q_{1A}+q_{2A})+p_{2A}(q_{3A}+q_{4A})\\&=\frac{1}{2}(p_{1A}+p_{2A})[(q_{1A}+q_{2A})+(q_{3A}+q_{4A})]+\frac{1}{2}(p_{1A}-p_{2A})[(q_{1A}+q_{2A})-(q_{3A}+q_{4A})]\\&=\frac{p_{1A}+p_{2A}}{4}+\frac{(p_{1A}-p_{2A})^2}{4t}\Delta\\&=\frac{t}{4\Delta}+\frac{k^2}{4t}\Delta.\end{aligned} \tag{70}$$

因为$x_1 = \frac{t-k}{2t}$,所以π_A只是k的函数。设k^*为求解一阶条件得到的k,那么当且仅当 $\pi_A(k^*)$ $< \pi^{U\text{-}U}$ 时，（D-$D2$）会降低利润。

参考文献

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